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13. ABSTRACT

Two probability distributions are currently used to set the load list quantity for submarine tenders. The Fleet Ballistic Missile (FBM) tender load list model uses the Poisson distribution if the quarterly average demand is less than or equal to one; otherwise, the Normal distribution is used. The conventional AS model uses the Normal distribution with a range cut (currently a quarterly average demand of .5) to set the load list quantity. There is little documentation to validate the use of either the Normal or Poisson distributions.

Summary statistics, box plots, and goodness-of-fit tests were used to evaluate the validity of the distributions currently in use and to hypothesize more distributions. Demand was analyzed at the aggregate, tender and item level. Since AS load lists are hull tailored, a test of homogeneity was conducted at the item level to determine if the same distribution can be used across all tenders. Load lists were built from the proposed distributions and compared to the benchmarks. Effectiveness was measured in terms of requisitions and units satisfied. The impact on range and cost was also evaluated.

We recommend computing the AS load list using the Geometric/Exponential probability distribution without a range cut in conjunction with a gross effectiveness goal. We further recommend that NAVSUPSYSCOM coordinate with SSPO to further evaluate the proposed distribution for FBM loads.

August 1986

AS DEMAND PROBABILITY DISTRIBUTION STUDY

OPERATIONS ANALYSIS DEPARTMENT

NAVY FLEET MATERIAL SUPPORT OFFICE
Mechanicsburg, Pennsylvania 17055

Report 165

AS DEMAND PROBABILITY
DISTRIBUTION STUDY

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REPORT 165

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ABSTRACT

Two probability distributions are currently used to set the load list quantity for submarine tenders. The Fleet Ballistic Missile (FBM) tender load list model uses the Poisson distribution if the quarterly average demand is less than or equal to one; otherwise, the Normal distribution is used. The conventional AS model uses the Normal distribution with a range cut (currently a quarterly average demand of .5) to set the load list quantity. There is little documentation to validate the use of either the Normal or Poisson distributions.

Summary statistics, box plots, and goodness-of-fit tests were used to evaluate the validity of the distributions currently in use and to hypothesize more appropriate distributions. Demand was analyzed at the aggregate, tender and item level. Since AS load lists are hull tailored, a test of homogeneity was conducted at the item level to determine if the same distribution can be used across all tenders. Load lists were built from the proposed distributions and compared to the benchmarks. Effectiveness was measured in terms of requisitions and units satisfied. The impact on range and cost was also evaluated.

We recommend computing the AS load list using the Geometric/Exponential probability distribution without a range cut in conjunction with a gross effectiveness goal. We further recommend that NAVSUPSYSCOM coordinate with SSPO to further evaluate the proposed distribution for FBM loads.

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EXECUTIVE SUMMARY

1. Background. The conventional submarine tender (AS) load list model uses the Normal distribution with a range cut (currently a quarterly demand average of .5) to set the load list quantity. The Fleet Ballistic Missile (FBM) tender load list model uses the Poisson distribution, if the quarterly average demand is less than or equal to one; otherwise, the Normal distribution is used. There is little documentation to support the use of either distribution.

2. Objective. To determine the appropriate distribution(s) for use in requirement's determination.

3. Approach. Forty-nine quarters of Mobile Logistics Support Force (MLSF) demand were used to hypothesize the appropriate distribution. The Navy Ships Parts Control Center (SPCC) provided a candidate file and a 90 day MLSF demand extract for the AS-11 to test the effectiveness of proposed probability distributions.

Summary statistics, box plots, and goodness-of-fit tests were used to evaluate the distributions currently in use and to hypothesize an appropriate distribution. Demand was analyzed at the aggregate and tender level to determine any patterns that exist, and at the item level to determine an appropriate probability distribution. Due to the hull constructed nature of AS load lists, a test of homogeneity was constructed at the item level to determine if the same distribution can be used across all tenders. Test loads were constructed using the proposed distributions and these alternative load lists were compared to actual demand. Effectiveness was measured in terms of requisitions and units satisfied. The impact on range and cost was also evaluated.

4. <u>Findings</u>. The homogeneity test indicated that item demand did not vary substantially among tenders having demand for that item. We found that demand is not Normally distributed at any level, and the Poisson distribution is also inappropriate. Demand is skewed in a rightward direction. The Geometric distribution provided the best fit for items with quarterly average demands less than or equal to one, and the Exponential distribution had the best fit for items with quarterly average demands greater than one.

Given the current AS gross requisition effectiveness, the Geometric/ Exponential produces a load with the same effectiveness as the current model, but 19% less cost. Given the current cost of an AS load, the Geometric/ Exponential produces a load with 4,954 more items and eight percentage points higher effectiveness. Given the same cost, the Geometric/Normal and Poisson/ Normal produce even higher range and effectiveness; 11,024 more items and 11 percentage points higher effectiveness. However, we feel that applying a budget goal is inappropriate for optimization models of this type. We were unable to assess the impact on AS(FBM) loads due to the numerous post-model adjustments made by VITRO.

5. <u>Conclusions</u>. AS tender demand is not Normally distributed. Instead, demand is extremely skewed in a rightward direction. The Geometric and Exponential provided the best fit for items with quarterly average demands less than or equal to one and greater then one, respectively. The Geometric/ Exponential distribution (with no range cut) is more cost effective than the current AS load. The Geometric/Normal and Poisson/Normal are also more cost effective than the current AS load and more cost effective than the Geometric/ Exponential at a budget goal; however, we feel that optimizing to a budget goal vice effectiveness goal is inappropriate and unrealistic.

6. Recommendation. We recommend that the conventional AS load list model use the Geometric/Exponential probability distribution with a gross effectiveness goal and no range cut instead of the current Normal distribution with a range cut of .5. We further recommend that NAVSUPSYSCOM coordinate with SSPO to evaluate the proposed probability distributions for an AS(FBM) and TRIDENT load list.

I. INTRODUCTION

Two probability distributions are currently used in computing Submarine Tender (AS) load lists. The Poisson distribution is used for Fleet Ballistic Missile tenders (AS(FBMs)) if the quarterly demand average is less than or equal to one. The Normal distribution is used for AS(FBMs) if the quarterly demand average is greater than one, and for conventional ASs, when the item passes the range cut (currently a quarterly demand average greater than or equal to 0.5). Range for AS(FBM) loads is determined by the depth computation; i.e., if the depth computes to zero, the item is not stocked.

The Normal distribution can be represented by a bell shaped curve. It is described by two parameters, the mean (μ) and the standard deviation (σ). The curve is symmetrical about μ, such that 68% of all observations lie within one σ of μ, 95.5% within two σ of μ, and 99.75% within three σ of μ.

Using the Normal distribution, μ and σ are computed as:

$$\mu = \left(\frac{1}{N}\right) \sum_{i=1}^{N} D_i$$

$$\sigma = \frac{1}{(N-1)} \left(\sum_{i=1}^{N} (D_i - \mu)^2 \right)^{\frac{1}{2}}$$

where

D_i = the demand for the i^{th} quarter

N = the number of quarters of demand (currently eight)

If historical demands (D_i) are all zero, then the Best Replacement Factor (BRF) forecast is used:

$$\mu = (BRF/4)(POP)$$

$$\sigma = \begin{cases} 1.6(\mu) & \text{for } \mu \geq 1 \\ 2.1(\mu) & \text{for } \mu < 1 \end{cases}$$

where

BRF = Best Replacement Factor

POP = total population of an item on the supported submarines that is Fleet installable and Tender installable

Next, the computed acceptable risk of stockout is used to compute the "t-value". The "t-value" represents the number of standard deviations away from the mean under the Normal curve that correspond to a specified risk.

The preliminary depth is then calculated as

$$\text{Load List Depth} = \mu + t\sigma$$

The Poisson distribution has a skewed (nonsymmetrical) shape. It is often used in counting the number of occurrences of an event of small probability in a fixed number of trials. It has one parameter λ. The value of λ is simultaneously the mean value and the variance of this distribution.

Using the Poisson distribution, the probability that demand equals zero is calculated first.

$$P(\text{Demand} = 0) = e^{-\lambda}$$

where

e = the base for the natural logarithm

λ = calculated same as the mean (μ) under the Normal distribution

If P(Demand = 0) is greater than or equal to the desired protection (1-risk), then the depth is set at zero. Otherwise, the probability that demand equals one is calculated.

$$P(\text{Demand} = 1) = \lambda\ (P(\text{Demand} = 0))$$

If the sum of P(Demand = 1) and P(Demand = 0) is greater than or equal to protection, we set the depth to one. Otherwise, we continue the process until the depth is found, using the following recursive formula:

$$P(\text{Demand} = X + 1) = P(\text{Demand} = X)\ \frac{\lambda}{X}$$

There is little documentation to validate the use of the Normal and Poisson distributions. The Simulation and Research System Policy and Concepts presentation for the retail models on 5 April 1983 established the need to determine the appropriate probability distribution(s) for use in load list requirements determination.

II. APPROACH

Mobile Logistics Support Force (MLSF) demand data from the second quarter 1972 to the second quarter 1984 for all submarine tenders and SUBASEs were used to hypothesize the appropriate probability distribution. The MLSF demand data were summarized in quarterly buckets. The Navy Ships Parts Control Center

(SPCC) provided a candidate file and a 90 day MLSF demand extract for the AS-11 covering the period from February 1985 to April 1985. These files were used to test the effectiveness of proposed probability distributions.

The following paragraphs describe the various statistical tests and performance measures used in the study. The mathematical formulae are found in APPENDIX A.

A. SUMMARY STATISTICS.

1. Measures of Centrality. There are three measures of location or central tendency of the data - mean, median, and mode. The mean $(\bar{X})$ is the average observation. The median (M) is the middle observation (midpoint) or 50th percentile. Half the observations are less than the median and half are greater than the median. The mode (M_o) is the most frequent observation.

The mean, median, and mode will be equal if the probability distribution is symmetric; e.g., the Normal distribution. If the distribution is skewed to the right, then the mean will be to the right of (greater than) the median. If the distribution is skewed to the left, then the mean will be to the left of (less than) the median.

2. Measures of Dispersion. The variance, standard deviation, and the variance to mean ratio are measures of the dispersion or spread of a distribution.

The variance to mean ratio (L) is particularly useful in determining discrete probability distributions; e.g., the Poisson, Geometric, and Binomial distributions. If the variance to mean ratio is greater than one, the distribution may be Geometric, equal to one implies Poisson, and less than one implies Binomial.

3. <u>Skewness</u>. Skewness (SK) measures the symmetry or shape of a distribution. If SK equals zero, the distribution is symmetric; e.g., Normal. When SK is positive, the distribution is skewed to the right; e.g., Poisson. If SK is negative, the distribution is skewed to the left.

4. <u>Kurtosis</u>. Kurtosis (K) measures the heaviness of the tails or height of a distribution. When most of the data are close to the mean, then K is negative. Therefore, flat distributions with short tails, such as the Uniform, have negative kurtosis. K is zero for the Normal distribution. Heavy tailed distributions, such as the Exponential, have positive kurtosis.

5. <u>Two Examples Using the Summary Statistics</u>. To demonstrate the value of these statistics, two data sets were generated using a random number generator. Data Set 1 was generated using the Normal distribution and Data Set 2 was created using the Poisson distribution. Each data set had 2,000 observations. TABLE I displays the summary statistics.

TABLE I

EXAMPLE SUMMARY STATISTICS

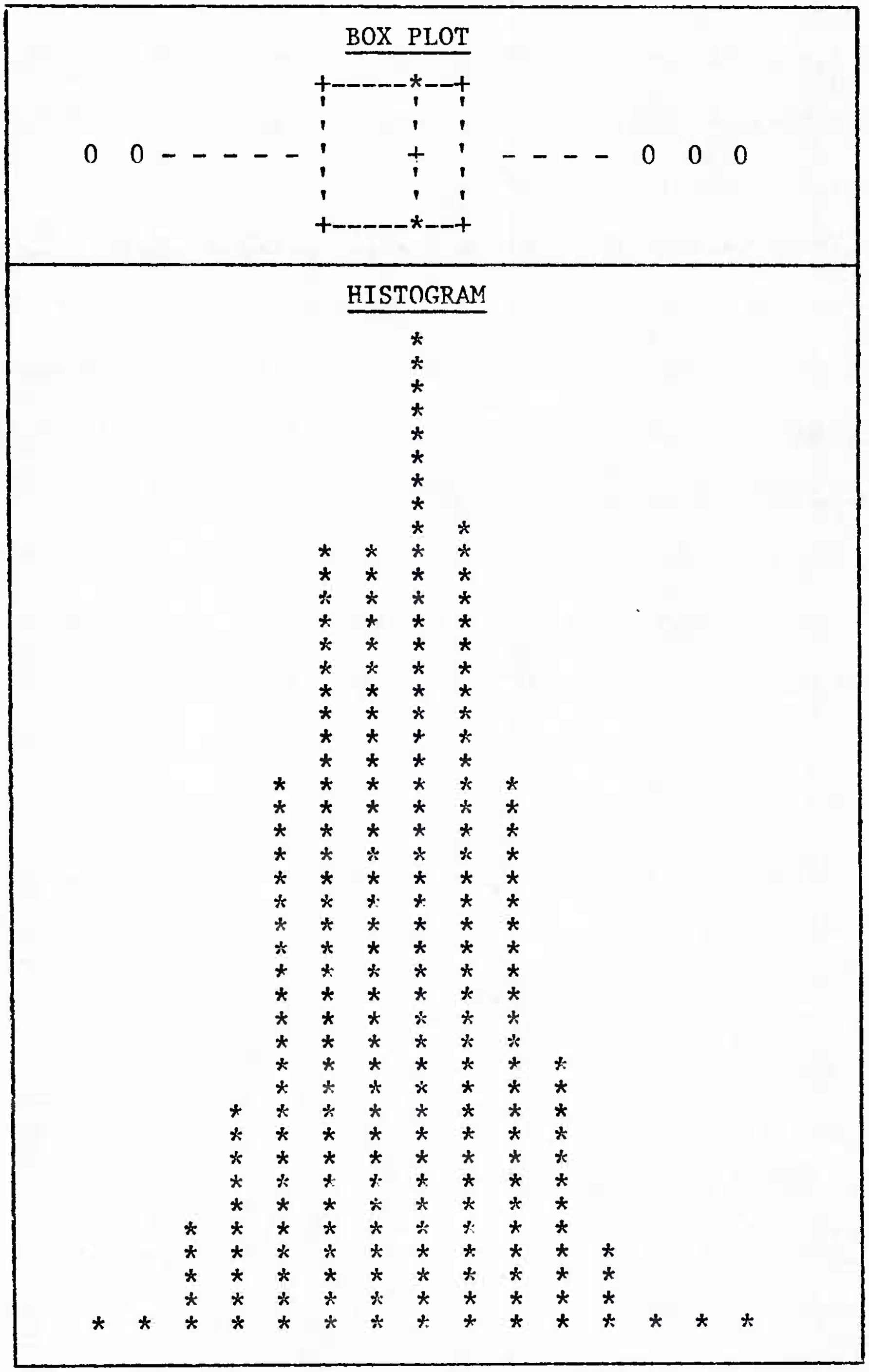
	DATA SET 1	DATA SET 2
Mean ($\overline{X}$)	-0.01	0.50
Median(M)	-0.04	0
Mode(M_o)	-3.51	0
Variance(S^2)	1.01	0.48
Std (S)	1.00	0.69
Variance to Mean Ratio (L)	-100.00	0.96
Skewness(SK)	- .01	1.28
Kurtosis(K)	- .11	1.17

The mean and median of Data Set 1 are fairly close (only .03 separates them). The mode is substantially different; however, the number of observations constituting the mode may not be significant. Both the skewness (SK) and the kurtosis (K) are close to zero. Therefore, based on the summary statistics, a symmetrical distribution, such as the Normal, would be a good candidate for Data Set 1.

The mean of Data Set 2 is greater than the median and mode, indicating the distribution is skewed in a positive direction. This is further validated by the positive value of SK (1.28). A positive value of K indicates heavy tail weight. Therefore, any skewed distribution, such as the Poisson, Binomial, Geometric, Exponential, or Chi-Square, would be a good candidate for Data Set 2. Since the variance to mean ratio is close to one, the Geometric distribution can be eliminated. Further analysis (goodness-of-fit tests, etc.) is needed to determine which distribution provides the best fit for both Data Set 1 and Data Set 2. The summary statistics only aid in the identification of candidate distributions.

B. <u>BOX PLOTS</u>. A box plot is a graphical display showing spread, skewness, tail length, and outlying data points. Box plots are particularly useful for comparing several data sets. The ends of the box plot, two vertical lines accentuated with plus signs (+), are located at the 25th and 75th percentiles. A vertical line, accentuated with asterisks (*), denotes the median. The mean is indicated by a plus sign (+). It is possible for the mean, median, and/or the 25th and 75th percentiles to have the same value (be on the same line). Horizontal lines - "whiskers" - denote data that are within 1.5 interquantiles of the box (where an interquantile is the length of the box; i.e., the distance between the 25th and 75th percentiles). More extreme values are represented by a zero (0) if they are within three interquantiles and an asterisk (*) if they are greater than three interquantiles.

Using the data from the previous examples, box plots for Data Set 1 and Data Set 2 are displayed in FIGURES 1 and 2, with their respective histograms. As can be seen, the box plots are more compact than the histograms, and box plots (through the use of symbols) better account for extreme data points. Due to its compact nature, it is easier to view several data sets simultaneously with box plots than with histograms. Furthermore, histograms are sensitive to the choice of intervals (or grouping factors). As can be seen by the following discussion, box plots yield more information than histograms.



BOX PLOT AND HISTOGRAM FOR EXAMPLE DATA SET 1

FIGURE 1

Using FIGURE 1, it is evident that Data Set 1 has a symmetrical shape by either graph. The box plot shows that the mean and median fall on the same line, and that the range of values between the 25th and 75th percentiles is small. Furthermore, approximately the same number of observations are smaller than the 25th percentile or are greater than the 75th, but no values lie outside of three interquantiles.

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BOX PLOT

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HISTOGRAM

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BOX PLOT AND HISTOGRAM FOR EXAMPLE DATA SET 2

FIGURE 2

The graphs in FIGURE 2 indicate that Data Set 2 is skewed. From the box plot of Data Set 2 we learn that the median and the 25th percentile have the same value, and the mean is larger than the median, which is expected when positive skewness occurs. (If negative skewness is the case, then we would expect the mean to be less than the median). Also, there are no values smaller than the 25th percentile, while there are values greater than the 75th. However, no values are larger than three interquantiles.

C. GOODNESS-OF-FIT TESTS. The summary statistics and box plots are useful for determining possible probability distributions. In order to determine which distribution has the best fit, we test the null hypothesis (H_o) that the data are representative of a particular distribution versus the alternative hypothesis (H_A) that the data are not of that distribution. If the test indicates that we should accept H_o, it does not necessarily mean that the data are represented by that particular distribution; but, that there is not enough evidence to reject the distribution.

In this study, three types of goodness-of-fit tests are used - the Kolmogorov-Smirnov (KS) test, the Shapiro-Wilk W Test, and the Chi-Square goodness-of-fit test. The KS test is valid for the Normal distribution and some other continuous distributions. The W test is a more appropriate test of the Normal distribution for small samples. The Chi-Square test is valid for all distributions, but is not considered as powerful. However, it has been shown to have good power against skewed distributions. Due to software and theoretical limitations, the KS test was used to test the Normal distribution if the number of observations exceeded 50, the W test was used for the Normal distribution if the number of observations was 50 or less, and the Chi-Square test was used for all other distributions. Each of the three goodness-of-fit tests is described below. The mathematical formulae are in APPENDIX A.

1. Shapiro-Wilk W Test. W is the ratio of the best estimate of the variance based on the sequence of a linear combination of the order statistics to the usual corrected sum of squares estimator of the variance (as defined in the summary statistics). W will have a value between zero and one. W is compared to the critical values given by Shapiro-Wilk.

2. Kolmogorov-Smirnov Test. Kolmogorov's D is computed by using a probability integral transformation function to convert data from the hypothesized distribution to the uniform distribution. A distance test is then made comparing the transformed data set to the uniform distribution, with large values of D leading to the rejection of H_o. The probability of observing a larger value of D is then computed and compared to a table of critical values.

3. Chi-Square Goodness-of-Fit Test. This technique tests the difference between the observed number of frequencies for an interval or cell, and the expected number of frequencies if the assumed distribution is correct. The resultant Chi-Square statistic (X^2) will have K-1-P degrees of freedom (where K equals the number of cells or intervals and P equals the number of estimated parameters for the distribution). If the hypothesized distribution provides a good fit for the data, we would expect the value for X^2 to be small.

D. KRUSKAL-WALLIS TEST FOR HOMOGENEITY. If we want to know whether or not data from two or more data sets are the same, then we conduct a test of homogeneity. The Kruskal-Wallis (K-W) test requires no knowledge or assumptions about the distribution of the data. It tests the null hypothesis (H_o) that the populations (data sets) are identical versus the alternative hypothesis (H_A)

that at least one of the populations (data sets) has larger values (observations) than at least one of the other data sets.

To construct the K-W statistic, we begin by merging the data sets and assigning ranks to all the observations, from 1 for the smallest value to N for the largest observation. In the event that two or more observations have the same value (tie), the ranks are averaged and each tied value is assigned the same rank. The ranks for each data set are then summed. The K-W statistic is then computed and adjusted for ties (KW'). We reject H_o if KW' is greater than or equal to a chi-square statistic (X^2) with K-1 degrees of freedom (where K is the number of data sets) at the appropriate level of acceptance.

E. PERFORMANCE MEASURES. Load list effectiveness was measured by comparing the test loads against 90 days of actual demands. Effectiveness was measured in terms of requisitions and units satisfied. The effectiveness measurements were made with noncandidate items included (gross effectiveness) and excluded (model effectiveness). Gross effectiveness represents the effectiveness the ship actually experiences, while model effectiveness indicates how well the math model determines range and depth for the candidate items. We also evaluated the impact on range and cost. The various performance measurements are defined below:

- Total Range = the number of items on the load list.
- Items on Load List without Demand = items on the load list that had no demand in the 90 day test period.
- Candidate Items not on Load List with Demand = items on the candidate file which did not make the load list, that had a demand in the 90 day test period.
- Cost of Load List = cost or value of the items on the load list.

- Value No Demand Items = cost or value of those items on the load list that had no demand in the 90 day test period.
- Gross Requisition Effectiveness = percent of all requisitions demanded in the 90 day test period, satisfied (both candidate and noncandidate items).
- Model Requisition Effectiveness = percent of requisitions demanded in the 90 day test period, and satisfied by those items that were on the candidate file (noncandidate items excluded).
- Gross Unit Effectiveness = percent of all units demanded in the 90 day test period, satisfied (both candidate and noncandidate items).
- Model Unit Effectiveness = percent of units demanded in the 90 day test period, and satisfied by those items that were on the candidate file (noncandidate items excluded).
- Predicted Model Effectiveness = Predicted number of requisitions satisfied divided by the predicted number of requisitions for candidate items.

III. FINDINGS

This section is comprised of two parts. The first part evaluates the validity of the probability distributions currently in use (the Normal and Poisson) and determines the appropriate probability distribution(s).
The second part discusses the effectiveness of loads built with the proposed probability distribution(s) versus the distributions currently in use.

A. <u>PROBABILITY DISTRIBUTION ANALYSIS</u>. First, we look at demand for the aggregate level. Next, we analyze demand for each AS. Then, we examine the demand for a sample of National Item Identification Numbers (NIINs). The intent is to determine what pattern(s) (if any) exist at the various levels of demand, in order to hypothesize a probability distribution.

1. Aggregate Level. The quantities demanded were summed quarterly for each NIIN across all tenders. TABLE II and FIGURE 3 display the summary statistics and box plot for the aggregate data set. As can be seen, the data are extremely skewed. The mean is 52 times greater than the median. Frequency distributions showed that the value for the 25th percentile is zero and the 75th percentile is seven. Ninety percent of the quantities demanded are 50 or less, which indicates that the mean is a value greater than the 90th percentile. The large measurements for the variance and standard deviation indicate that the aggregate data have a large spread. However, the distance between 25th and 75th percentiles is very small, as evidenced by the same line on the box plot. There are no values smaller than the 25th percentile, but many values more than three interquantiles greater than the box. The values for the skewness and kurtosis are extreme. Based on the summary statistics and box plot, we would not be willing to believe that demand, in the aggregate, is Normally distributed. The Kolmogorov's D statistic, .48, and the .01 probability of observing a D statistic that size, validate this assumption. The variance to mean ratio makes the Poisson distribution an unlikely candidate since its value is much greater than one.

TABLE II

SUMMARY STATISTICS FOR THE AGGREGATE LEVEL

Mean ($\bar{X}$)	52
Median (M)	1
Mode (M_o)	0
Variance (S^2)	1,526,692
Std (S)	1,235
Skewness (SK)	112
Kurtosis (K)	15,989
Variance to Mean Ratio	29,357
D	.48
$P \geq D$	.01

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BOX PLOT OF AGGREGATE DATA

FIGURE 3

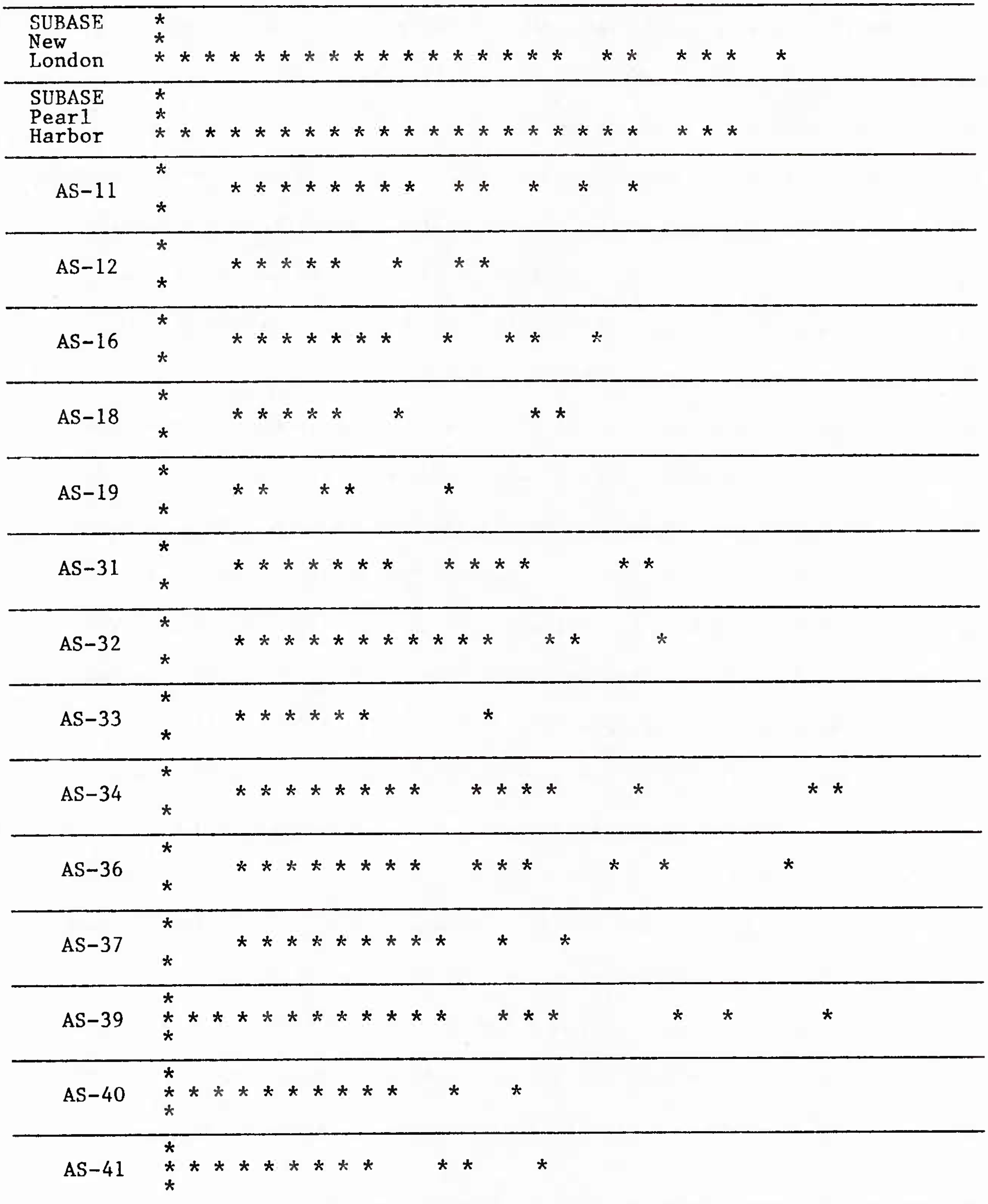
2. <u>Tender Level</u>. TABLE III and FIGURE 4 show summary statistics and box plots by Tender/SUBASE and indicate that demand at the tender level has a similar pattern to demand at the aggregate level. There are no observations less than the 25th percentile. The 25th percentile, the median, and 75th

percentile all have the same value, zero, for each Unit Identification Code (UIC). In each case, the mean is larger then the median. While the degree of skewness varies from tender to tender, every tender's demand is extremely skewed. The measurements of skewness and kurtosis at the tender level are consistent with those found at the aggregate level, as are the D statistics and the probabilities of observing D statistics that size. The variance to mean ratio was not computed for each UIC; however, the variance in each case is much larger than the mean and the ratios would all be greater than one. Thus, neither the Poisson nor Normal distributions are good candidates for demand at the tender level.

TABLE III

SUMMARY STATISTICS FOR INDIVIDUAL TENDERS

UIC	$\overline{X}$	M	M_o	S^2	S	SK	K	D	P > D
N.London	4.8	0	0	34,238	185	147	27,092	.49	.01
P. Harbor	5.5	0	0	24,255	155	90	9,673	.49	.01
AS-11	4.9	0	0	16,218	127	138	28,296	.48	.01
AS-12	4.1	0	0	11,787	108	132	24,643	.49	.01
AS-16	5.1	0	0	36,196	190	105	13,394	.49	.01
AS-18	5.6	0	0	42,953	207	185	52,419	.49	.01
AS-19	4.1	0	0	19,290	138	166	44,603	.49	.01
AS-31	5.6	0	0	20,926	144	119	19,826	.48	.01
AS-32	6.1	0	0	24,895	157	109	16,222	.48	.01
AS-33	6.6	0	0	32,696	180	135	28,557	.49	.01
AS-34	6.2	0	0	20,925	144	156	39,898	.48	.01
AS-36	5.5	0	0	22,810	151	137	26,629	.49	.01
AS-37	4.8	0	0	18,860	137	112	17,025	.49	.01
AS-39	4.4	0	0	8,860	94	108	18,412	.48	.01
AS-40	4.4	0	0	10,143	100	119	20,604	.48	.01
AS-41	3.0	0	0	8,181	90	161	36,332	.49	.01



BOX PLOTS FOR INDIVIDUAL TENDERS

FIGURE 4

3. <u>NIIN Level</u>. Thus far, we have confined ourselves to demand at aggregate or tender levels. This analysis was useful in determining the patterns that exist; but to determine an appropriate probability distribution, it is necessary to consider individual items.

We took a sample of one hundred items using a Normal random number generator. In order to test the validity of the current distributions (Normal and Poisson) and hypothesize a new distribution, we decided that an item needed at least five positive quarterly demands. Twenty-five of the hundred items met this requirement. The sample is validated in APPENDIX B.

AS load lists are hull constructed (i.e., a selected group of submarines are supported by each tender). Since load lists and demands vary from tender to tender, each tender could have its own demand distribution. Therefore, we conducted the K-W test for the quarterly demands for an item across all tenders having demand for that item to determine if we can apply the same probability distribution to all tenders. The Chi Square statistics, degrees of freedom, and associated probabilities of observing a Chi Square (KW') statistic that size are displayed in TABLE IV. The test could not be conducted for one item and was unnecessary for four others (the data were the same on each tender or there was only one tender having demand for that item). Of the items tested, only one was found to have demand not homogeneous across tenders having demand for that item (as the probability of observing a KW' statistic sheet size (.0001 was less than .01). Since demand for an item did not vary substantially among the tenders, the quarterly demands were combined across the ASs, and it was not necessary to fit a probability distribution for each tender.

TABLE IV

KRUSKAL-WALLIS TEST OF HOMOGENEITY

NIIN	X^2	Degrees of Freedom	Probability > KW'
000014199	21.08	14	0.0996
000014937	2.35	6	0.8844
000016482	6.78	8	0.5608
000016503	1.24	2	0.5388
000018027	Number of UICs greater than number of observations		
000018039	2.00	2	0.3679
000019359	9.35	7	0.2282
000030295	3.13	7	0.8724
000030688	3.00	4	0.5578
000034011	1.50	3	0.6823
000035490	38.42	10	0.0001
000035845	7.81	9	0.5534
000042695	All data tied		
000043365	Only one UIC		
000044486	12.01	11	0.3628
000044489	17.25	12	0.1406
000048237	All data tied		
000049138	12.00	9	0.2130
000050592	4.35	4	0.3612
000066405	1.50	3	0.6823
000085153	All data tied		
000096804	8.68	9	0.4671
000105365	5.00	3	0.1718
000105602	7.70	10	0.6579
000120809	23.39	14	0.0543

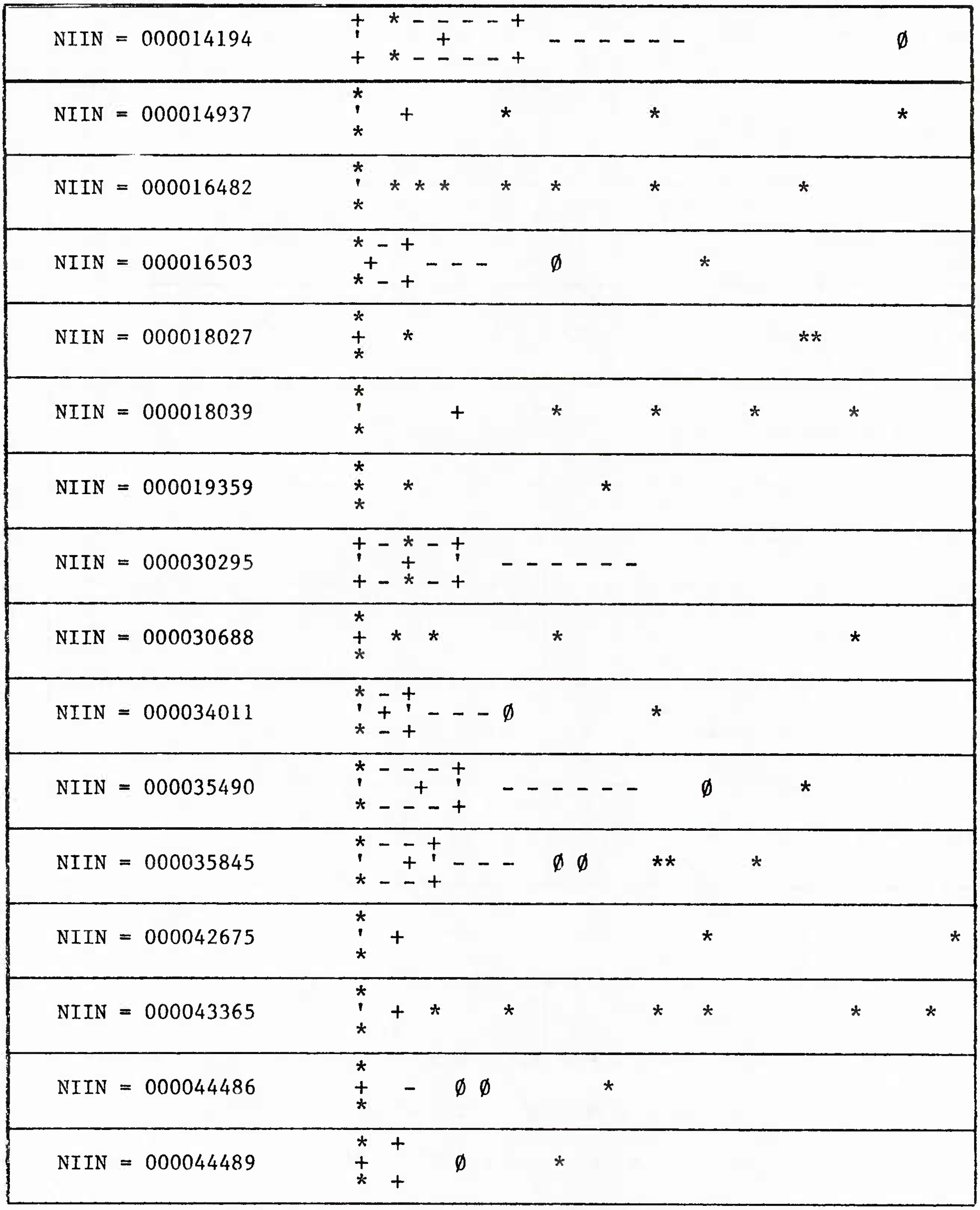
The summary statistics in TABLE V, and box plots in FIGURE 5, indicate that every item is skewed in a positive or rightward direction, which is consistent with demand at the aggregate and tender levels. However, the degree of positive skewness and spread for each item as viewed by the box plots varies. The degree of skewness (SK) ranges from 0.9 to 6.6. Kurtosis (K) was measured from - 0.2 to 44.7. Therefore, it is unlikely that any distribution can be found that fits demand for every item. However, it is unlikely that any item in our data is Normally distributed.

TABLE V

ITEM SUMMARY STATISTICS

NIIN	$\overline{X}$	M	M_o	S^2	S	SK	K	L
000014194	32.2	16	0	1,389.9	37.3	1.7	3.3	-
000014939	0.4	0	0	0.9	0.9	2.8	7.8	2.1
000016482	2.1	0	0	43.1	6.6	4.0	17.0	-
000016503	2.6	0	0	25.0	5.0	2.9	9.8	-
000018027	0.9	0	0	16.8	4.1	4.8	21.7	19.2
000018039	0.4	0	0	0.9	1.0	2.8	7.7	2.2
000019359	13.9	0	0	5,271.48	72.6	6.6	44.7	-
000030295	1.4	1	0	2.5	1.6	0.9	-0.2	-
000030688	1.2	0	0	23.7	4.9	5.8	35.4	-
000034011	0.3	0	0	0.5	0.7	2.5	6.2	1.5
000035490	13.4	0	0	406.8	20.2	2.0	5.1	-
000035845	2.1	0	0	16.8	4.1	2.4	5.4	-
000042675	0.1	0	0	0.2	0.4	3.4	12.1	1.2
000043365	0.7	0	0	4	2	3.1	9.1	5.6
000044486	442.0	250	0	475,029	689.2	4.0	20.6	-
000044489	449.1	250	0	663,303	814.4	4.6	26.7	-
000048237	0.3	0	0	0.4	0.7	1.7	1.6	1.3
000049138	0.7	0	0	1.2	1.1	2.1	4.8	1.9
000050592	1.6	0	0	18.4	4.3	3.7	15.8	-
000066405	0.2	0	0	0.3	0.6	2.4	5.0	1.3
000085153	0.6	0	0	0.7	0.8	1.3	0.5	1.2
000096804	5.6	0	0	202.1	14.2	5.0	29.4	-
000105365	0.2	0	0	0.3	0.6	3.9	16.4	1.9
000105602	5.0	4	4	21.4	4.6	1.2	.9	-
000120809	124.8	111	63	7,903.9	88.9	1.8	4.1	-

NOTE: Since the Poisson distribution is only used for those items with means less than one, we computed the variance to mean ratio for those items to test the necessary condition for the Poisson distribution; i.e., L equals one.



ITEM BOX PLOTS

FIGURE 5

```
                           * - - - - +
NIIN = 000048237           '   +       - - - - -              Ø
                           * - - - - +

                           * - +
NIIN = 000049138           '  +      - -        Ø   *
                           * - +

                           *
NIIN = 000050592           * * * * * * *                     *
                           *

                           *
NIIN = 000064405           '       +              *               *
                           *

                           * - - - - +
NIIN = 000085153           '   +     ' - - - - -                  Ø
                           * - - - - +

                           * - +
NIIN = 000096804           '   + - Ø Ø     *                              *
                           * - +

                           *
NIIN = 000105365           +           *           *           *
                           *

                           + - - * - - +
NIIN = 000105602       -   '     +     ' - - - - - - - - -   Ø Ø Ø
                           + - - * - - +

                           + - - * - - +
NIIN = 000120809       - -       '  +  '   - - - - - - -     Ø
                           +     *     +
```

ITEM BOX PLOTS

FIGURE 5 (CONT)

The Shapiro-Wilkes test for Normality, shown in TABLE VI, confirms this assumption. No item was able to pass this test since the probability of observing a W statistic of this size was less than .01. Furthermore, the consistent positive value for the Variance to Mean Ratio (L), shown in TABLE V for those items with quarterly average demands of one or less, indicates that the Poisson distribution may not be a valid choice for those items.

TABLE VI

SHAPIRO-WILKS TEST

NIIN	'W' Test	Probability > W
000014194	0.814067	< 0.01
000014937	0.506876	< 0.01
000016482	0.38581	< 0.01
000016503	0.608417	< 0.01
000018027	0.227411	< 0.01
000018039	0.492614	< 0.01
000019359	0.208746	< 0.01
000030295	0.79725	< 0.01
000030688	0.278799	< 0.01
000034011	0.539846	< 0.01
000035490	0.720658	< 0.01
000035845	0.592723	< 0.01
000042675	0.365203	< 0.01
000043365	0.421453	< 0.01
000044486	0.613846	< 0.01
000044489	0.540323	< 0.01
000048237	0.55889	< 0.01
000049138	0.645646	< 0.01
000050592	0.450605	< 0.01
000066405	0.458841	< 0.01
000085153	0.689115	< 0.01
000096804	0.436158	< 0.01
000105365	0.349079	< 0.01
000105602	0.859773	< 0.01
000120809	0.833942	< 0.01

At this point, we can conclude that demand is not Normally distributed at any level. In fact, due to the skewness of demand, the Normal distribution would be a poor choice to model AS demand. Recall from our earlier discussion of the summary statistics, we stated that a Normal distribution's mean would be at the 50th percentile; i.e., the mean and median would be equal. (That is, we would expect half the population to be larger and half smaller than the mean). As indicated by the box plots in FIGURE 5 and the summary statistics in TABLE V, the mean is consistently at some level greater than the median (50th percentile). Therefore, the load list quantity will be set at a different level than prescribed by the risk for that item; i.e., the level of protection and budget will be set at different goals (effectiveness) than computed by the model. We need a distribution which can account for the positive skewness.

The following distributions were selected as candidates and tested by the Chi-Square goodness-of-fit test for each sample item:

Binomial - Explains the positive skewness but unlikely to explain the high variability of demand.

Geometric - A special case of the Negative Binomial distribution. Like the Negative Binomial, it explains positive skewness and a high degree of variability, but handles zero values better.

Poisson - Tested because it is currently in use.

Gamma - Explains positive skewness.

Chi-Square - A special case of the Gamma distribution.

Exponential - A special case of the Gamma distribution and continuous analog of the Geometric distribution. Explains positive skewness. Previously found to describe demand in various retail and wholesale situations.

TABLE VII summarizes the Chi-Square goodness-of-fit test results for the sample items.

TABLE VII

CHI-SQUARE GOODNESS-OF-FIT TEST

	Binomial	Geometric	Poisson	Gamma	Chi-Square	Exponential
000014194		X				X
000014937						
000016482						
000016503						
000018027						
000018039						
000019359						
000030295		X				
000030688						
000034011						
000035490						
000035845						
000042675						
000043365						
000044486						X
000044489						
000048237						
000049138		X				
000050592						
000066405						
000085153	X	X	X			
000096804						
000105365						
000105602		X			X	
000120809		X		X		X

Legend: X denotes a good fit.

Only 13 of the tests indicated a good fit at a significance level of 0.01. Only the Geometric and Exponential distributions had good fits for more than one test. (The Chi-Square statistics, degrees of freedom, and probability of observing a Chi-Square statistic that size can be found in APPENDIX C.) Of the six good fits for the Geometric distribution, three of the items had quarterly demand averages less than one, and one was close to one (1.4). The remaining two NIINs had means greater than one and also had a good fit for the Exponential distribution. The three items for which the Exponential distribution was a good fit all had means greater than one.

B. PROBABILITY DISTRIBUTION EVALUATION. As stated in the approach, two distributions are currently used in submarine tender load list computations to model demand. Conventional tenders load list computations use the Normal distribution with a range cut of .5 (the conventional benchmark). FBM tenders load list computations use the compound Poisson/Normal distribution to set range and depth (FBM Model).

The preceding analysis indicated that the Geometric distribution was a good candidate for items with quarterly demand averages less than or equal to one, and the Exponential distribution was a good candidate for items with quarterly average demands greater than one. Both the Geometric and Exponential distributions are very easy to use. Using the Geometric cumulative probability distribution and solving it algebraically for the load list quantity yields:

$$Q = (\ln \text{Risk}/\ln (1-p)) - 1$$

where

Q = load list quantity

ln = the natural log

$p = 1/(\mu + 1)$

μ = the quarterly demand average

Using the Exponential cumulative probability distribution and solving algebraically for the load list quantity yields:

$$Q = -\mu \ln \text{risk}$$

where

Q = the load list quantity

ln = the natural log

μ = the quarterly demand average

Various combinations of the current distributions and the Geometric and Exponential distributions were tested with and without a range cut. Several past studies indicated that the probability of a demand can be modeled by the compound Bernoulli/Exponential distribution. These studies indicate that if an item had no past demand, it can be modeled by the Bernoulli distribution. If an item had past demand, it could be represented with the Exponential distribution. As the load list includes both demand and BRF items, (items with no historical demand in the candidate data base), we adapted the Bernoulli/Exponential distribution.

To test this distribution, we considered the eight quarters of demand as the number of events, and a positive value of quarterly demand as a success. The probability of a success is then equal to the quarterly demand average (μ). If μ is greater than or equal to one, we compute the load list quantity using the Exponential distribution, as described previously. Otherwise, we use the Bernoulli distribution to compute the probability (1-) that demand would equal zero. Then, we compare the probability of a zero demand with the required protection (1-risk). If the probability of a zero demand is greater than the protection, we set the load list quantity to zero. Otherwise, we compute the load list quantity using the Exponential distribution. Thus, the Bernoulli distribution acts as a variable range cut.

We built the test loads to a predicted net requisitions effectiveness goal, a predicted gross requisitions effectiveness goal, and a budget goal. The results for the net effectiveness goal runs are shown in TABLE VIII.

The Normal distribution with the Geometric and Poisson distribution (without a range cut) had undesirable results in terms of range and effectiveness. We got this result since a net effectiveness goal is not appropriate for a model with no range cut; i.e., one item could conceivably provide the required net effectiveness.

Replacing the benchmark Normal distribution with the Exponential distribution, while retaining the current range cut of .5, produced little difference. The Exponential distribution did provide higher requisitions effectiveness than the benchmark, but provided less units effectiveness. The cost for the load lists were similar.

The Exponential distribution, in conjunction with either the Geometric, Bernoulli, or Poisson distributions (without a range cut), generated similar load lists with respect to effectiveness and cost. However, we prefer the Geometric/Exponential since it is the easiest to use and the only one theoretically defensible.

TABLE VIII

EFFECTIVENESS STATISTICS
(85% NET EFFECTIVENESS GOAL)

	Normal Range Cuts (Benchmark)	Poisson Normal	Geometric Normal	Exponential Range Cut .5	Geometric Exponential	Bernoulli Exponential	Poisson Exponential
Total Range	17,718	2,971	2,947	17,718	16,248	17,193	16,557
Items on Load List with Dmd	4,358	390	389	4,358	3,904	4,040	3,937
Items on Load List w/o Dmd	13,360	2,581	2,558	13,360	12,344	13,153	12,620
Candidate Items Not on Load List with Demand	986	4,956	4,957	986	1,441	1,305	1,408
Cost of Load List	$5.7M	$2.6M	$2.6M	$5.9M	$4.4M	$4.5M	$4.4M
Value No Demand Items	$4.3M	$2.3M	$2.3M	$4.4M	$3.2M	$3.3M	$3.2M
Gross Requisition Eff	62.4%	6.6%	6.6%	63.4%	59.6%	60.5%	59.9%
Model Requisition Eff	74.8%	8.0%	8.0%	76.0%	71.4%	72.5%	71.8%
Gross Units Eff	59.9%	17.5%	17.5%	57.4%	56.6%	56.5%	56.7%
Model Units Eff	73.7%	21.5%	21.5%	70.7%	69.7%	69.6%	69.8%
Predicted Model Eff	72.1%	31.5%	31.5%	72.2%	69.2%	70.0%	69.5%

NOTE: Where two distributions are shown, the first was used for items with an average quarterly demand of one or less, and the second was used for items with an average quarterly demand greater than one. Unless a range cut is specified, the range was determined by the depth computation.

Comparing the Geometric/Exponential to the conventional AS benchmark (Normal with a .5 range cut) the Geometric/Exponential decreases effectiveness 2.8 - 4.0 percentage points. However, there is a large difference in cost. The Geometric/Exponential costs 22% less than the conventional benchmark. Most of the savings comes from not stocking items which had no demand during the evaluation period. The cost of items having no demand during the evaluation period is 26% less for the Geometric/Exponential than for the benchmark.

As explained in the beginning of this section, there is some question about the validity of building a load list to a predicted net effectiveness goal without a range cut, since we could compute a predicted net effectiveness of 85% for a load list with one item. Therefore, we ran several of the alternatives to the same budget and then the same predicted gross effectiveness as the benchmark. We feel that optimizing to a budget goal is inappropriate and unrealistic. Optimizing to a performance goal such as predicted gross effectiveness is more appropriate and manageable. We considered a budget goal only for comparison purposes.

TABLE IX summarizes results at the same budget level. Higher range and higher effectiveness was achieved in every category for the models without a range cut as compared to the computational AS benchmark. The Geometric/Exponential and Bernoulli/Exponential distributions had similar results, as did the Poisson/Normal and Geometric/Normal distributions. Of the models without a range cut, those using the Normal Distribution for the higher demand items had a slightly higher effectiveness than did the Exponential models. However, the cost of items with no demand was about 10% higher. Furthermore, the Normal models stocked about 25% more items than the Exponential models.

TABLE IX

EFFECTIVENESS STATISTICS
$5.7M BUDGET GOAL

	Normal Range Cut .5 (Conv. Benchmark)	Geometric Exponential	Bernoulli Exponential	Geometric Normal	Poisson Normal (FBM Model)
Total Range	17,718	22,672	23,520	28,456	28,742
Items on Load List with Dmd	4,358	4,314	4,390	4,529	4,549
Items on Load List w/o Dmd	13,360	18,358	19,180	23,927	24,193
Candidate Items Not on Load List with Demand	986	1,031	955	817	797
Cost of Load List	$5.7M	$5.6M	$5.7M	$5.7M	$5.7M
Value No Demand Items	$4.3M	$3.6M	$3.7M	$4.0M	$4.0M
Gross Requisition Eff	62.4%	69.4%	70.0%	72.0%	72.0%
Model Requisition Eff	74.8%	83.2%	83.9%	86.3%	86.3%
Gross Units Eff	59.9%	62.8%	62.8%	67.3%	67.2%
Model Units Eff	73.7%	77.3%	77.3%	82.8%	82.7%
Predicted Model Eff	72.1%	81.5%	82.3%	87.3%	87.4%

NOTE: Where two distributions are shown, the first was used for items with an average quarterly demand of one or less, and the second was used for items with an average quarterly demand greater than one. Unless a range cut is specified, the range was determined by the depth computation.

TABLE X summarizes results of building the loads to the same gross effectiveness goal as the benchmark. Again, the models using the Exponential distribution had similar results, as did the Normal models. Here, the Normal models, without a range cut, provided less range and about four percentage points less requisition effectiveness than either the Exponential models or the benchmark. However, they did provide better unit effectiveness. Both Exponential models provided the same range and level of effectiveness as the benchmark, but cost 19% less. The cost of items on the load list with no demand was 23% less for the Exponential models than for the benchmark. As stated previously, the Geometric/Exponential is preferred over Bernoulli/ Exponential since it is theoretically justified in the earlier part of this study and is easier to use.

TABLE X

EFFECTIVENESS STATISTICS
72% GROSS EFFECTIVENESS GOAL

	Normal Range Cut .5 (Conv. Benchmark)	Geometric Exponential	Bernoulli Exponential	Geometric Normal	Poisson Normal (FBM Model)
Total Range	17,718	17,261	17,902	15,191	15,394
Items on Load List with Dmd	4,358	3,983	4,091	3,398	3,416
Items on Load List w/o Dmd	13,360	13,278	13,811	11,793	11,978
Candidate Items Not on Load List with Demand	986	1,362	1,254	1,948	1,930
Cost of Load List	$5.7M	$4.6M	$4.6M	$3.5M	$3.5M
Value No Demand Items	$4.3M	$3.3M	$3.3M	$2.7M	$2.7M
Gross Requisition Eff	62.4%	62.3%	62.4%	58.6%	58.6%
Model Requisition Eff	74.8%	74.7%	74.8%	70.3%	70.3%
Gross Units Eff	59.9%	58.5%	58.0%	62.0%	61.9%
Model Units Eff	73.7%	72.0%	71.5%	76.3%	76.2%
Predicted Model Eff	72.1%	72.0%	72.1%	72.0%	72.1%

NOTE: Where two distributions are shown, the first was used for items with an average quarterly demand of one or less, and the second was used for items with an average quarterly demand greater than one. Unless a range cut is specified, the range was determined by the depth computation.

The preceding discussion focused on the conventional AS tender benchmark. We were unable to construct an FBM benchmark since VITRO and the Strategic Systems Project Office (SSPO) make numerous post-model changes to the FBM load list. However, the current FBM policy of using the Poisson/Normal distribution can be evaluated by comparing the FBM model (Poisson/Normal distribution) on AS-11 data, to the Geometric/Exponential, Bernoulli/ Exponential, and Geometric/Normal at the same budget and same predicted gross effectiveness goals, as discussed previously and depicted in TABLES IX and X.

Here, the results are mixed. At the budget goal the Poisson/Normal provided greater range and effectiveness and had more nonmovers than the Geometric/Exponential. At the predicted gross effectiveness goal, the Geometric/Exponential distribution had greater range and requisition effectiveness but less unit effectiveness than the FBM model. The Geometric/ Exponential also cost more and had more nonmovers. For both goals, the actual model effectiveness (percent of requisitions satisfied for all candidates) was greater than the predicted model effectiveness for the Geometric/Exponential distribution and less than the predicted model effectiveness for the Poisson/ Normal distribution; i.e., the Geometric/Exponential underestimates effectiveness while the Poisson/Normal overestimates effectiveness.

IV. SUMMARY AND CONCLUSIONS

The results indicate that AS historical demand is not Normal at the aggregate level, tender level, or item level. Demands are positively skewed at every level. The degree of skewness and spread varies from item to item; thus, it is difficult to determine one distribution which fits demand for every item.

The Geometric probability distribution provides the best fit for items with quarterly demand averages less than or equal to one. The Exponential probability distribution provides the best fit for items with quarterly demand averages greater than one. Both the Geometric and Exponential distributions are faster and easier to calculate (and thus less costly) than the other distributions considered.

Testing the effectiveness of the probability distributions indicates that different distributions appear more cost effective when run at different goals. We showed that applying a net effectiveness goal to a model with no range cut is not appropriate. Applying a budget goal to an optimization model of this type is also inappropriate. A budget goal will not guarantee a specified performance level nor will it guarantee the same level of performance for subsequent computations. A predicted gross effectiveness goal is the best choice for this type of optimization model, as a gross effectiveness goal provides to the user a more realistic and manageable method to build a load list. Considering range, cost, effectiveness, and the preceding discussion of appropriate goals, the Geometric/Exponential appears to be the best alternative for conventional ASs. At the same predicted gross effectiveness as the current distribution (Normal with a range cut), the Geometric/Exponential costs 19% less while providing the same requisition effectiveness.

Due to the numerous post-model changes made to the FBM loads, we were unable to quantify the impact of applying the Geometric/Exponential to FBM loads. However, the Geometric/Exponential distribution was shown to provide better effectiveness than the Poisson/Normal when run to a gross effectiveness goal.

V. RECOMMENDATIONS

We recommend computing the AS load list using the Geometric/Exponential probability distribution without a range cut in conjunction with a gross effectiveness goal. We further recommend that NAVSUPSYSCOM coordinate with SSPO to further evaluate the proposed distribution for FBM loads.

APPENDIX A: SUMMARY STATISTICS/GOODNESS-OF-FIT FORMULAE

The Mean.

$$\overline{X} = \frac{1}{N} \sum_{i=1}^{N} X_i$$

where

X_i = the i^{th} observation

N = the number of observations

The Median.

The sample median of $[X_1, X_2, \ldots, X_n]$ is

$$M = \begin{cases} X_{(N+1)/2} & \text{if N is an odd number} \\ \frac{1}{2}(X_{N/2} + X_{((N/2)+1)}) & \text{if N is an even number} \end{cases}$$

The Variance.

$$S^2 = \frac{1}{N-1} \sum_{i=1}^{N} (X - \overline{X})^2$$

where

X_i = the i^{th} observation

X = the mean

N = the number of observations

The Standard Deviation.

$$S = (S^2)^{1/2}$$

where

S^2 = the variance

The Variance to Mean Ratio (L):

$$L = S^2/\overline{X}$$

where

S^2 = the variance

$\overline{X}$ = the mean

Skewness.

$$SK = [N/(N-1)(N-2)] \sum_{i=1}^{N} (X_i - \overline{X})^3/S^3$$

where

N = the number of observations

X_i = the i^{th} observation

$\overline{X}$ = the mean

S = the standard deviation

Kurtosis.

$$K = \left[[(N(N+1)/(N-1)(N-2)(N-3)] \sum_{i=1}^{N} (X_i - \overline{X})^4/S^4 \right] - 3(N-1)^2/(N-2)(N-3)$$

where

N = the number of observations

X_i = the i^{th} observation

$\bar{X}$ = the mean

S = the standard deviation

Shapiro-Wilk.

$$W = b^2/S^2$$

where

S^2 = the variance

$$b = \sum_{i=1}^{k} a_{n-i+1}(X_{n-i+1} - X_i)$$

where

n = the number of observations in the data set

$$k = \begin{cases} N/2 \text{ if } N \text{ is even} \\ (N-1)/2 \text{ if } N \text{ is odd} \end{cases}$$

X_i = the i^{th} observation

the values of a_{n-i+1} are provided in a table by Shapiro.

Kolmogorov's D.

$$D = \sqrt{N} \left(\max_{1 \leq R \leq N} |R/N - F(X_R)| \right)$$

where

R = the rank of the observation (R = 1, 2, ..., N)

$$X_1 \leq X_2 \leq \ldots \leq X_N$$

N = the number of observations

$F(X_R)$ = the value of the transformed observation

<u>Probability of Observing a Larger Value of D.</u>

$$(N-.01 + .85/\sqrt{N})^{\frac{1}{2}} \ D$$

where

N = the number of observations

D = the D statistic (test statistic)

<u>Chi-Square.</u>

$$X^2 = \sum_{i=1}^{K} (Fo_i - Fe_i)^2/Fe_i$$

where

Fo_i = the number of observed frequencies for the i^{th} cell or interval

Fe_i = the number of expected frequencies for the i^{th} cell or interval

K = the number of cells or intervals

<u>K-W.</u>

$$K\text{-}W = H = \left[\frac{12}{N(N+1)}\right]\left[\sum_{i=1}^{K} R_i{}^2/n_i\right] -3(N+1)$$

where

N = the total number of observations

n_i = the number of observations for the i^{th} data set

R_i = the sum of the ranks for the i^{th} data set

K = the number of data sets

$KW' = H/C$

where

$$C = 1 - \sum_{i=1}^{\ell} (t_i^3 - t_i)/(N^3 - N)$$

where

t_i = the rank assigned the i^{th} tied items

N = the total number of observations

ℓ = the number of ties

APPENDIX B: SAMPLE VALIDATION

Due to software limitations, we were not able to use the K-W test to validate the sample (the population was too large). The skewness makes the F test and T tests inappropriate (we are not able to justify the assumption that the values are Normally distributed around the mean.) Therefore, TABLE I will compare the summary statistics between the sample and the total population.

TABLE I

SUMMARY STATISTICS

	SAMPLE	POPULATION
Minimum	0	0
25th Percentile	0	0
Median	1	1
75th Percentile	8	7
Mean	70.2	52.0
Mode	0	0
P(0)	.49	.48
S^2	119,340	1,526,692
S	345.5	1,235.6
Skewness	10.6	112.0
Kurtosis	138.5	15,989.6
Max	53,000	304,414

The sample and total population are the same through the 75th percentile. The probability of obtaining a zero value, and the modes, are the same for both data sets. The difference is in the skewness. The population is more skewed than the sample. The maximum value for the population is approximately six times greater then the maximum value observed in the sample. Thus, the variance, standard deviation, and mean are larger for the population than for the sample. This is not a cause for concern. When a population is skewed, then a sample drawn from that population will generally be less skewed than the population. In this case, 75% of the observations will be less than or equal to seven. Thus, we have a better chance (there is a higher probability) of observing values less than or equal to seven than values greater than seven. As values become more extreme, the chance of observing them in the sample becomes smaller. Given this condition, the sample appears to be a good representation of the population.

APPENDIX C: GOODNESS-OF-FIT TEST STATISTICS

TABLE VII

GOODNESS-OF-FIT TEST STATISTICS
(SAMPLE NIINs)

NIIN	Distribution	X^2	df	$P > X^2$
000014194	Geometric	12.21	2	.094
000014194	Exponential	7.02	2	.030
000030295	Geometric	4.29	2	.119
000044486	Exponential	5.85	1	.016
000049138	Geometric	2.95	1	.086
000085153	Binomial	4.08	1	.043
000085153	Geometric	1.39	1	.238
000085150	Poisson	3.94	1	.047
000105602	Geometric	6.44	5	.266
000105602	Chi Square	4.78	3	.188
000120809	Geometric	17.01	7	.017
000120809	Exponential	6.78	3	.079
000120809	Gamma	2.56	2	.277

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